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1990 J. Phys. A: Math. Gen. 23 2147

(http://iopscience.iop.org/0305-4470/23/11/035)

J. Phys. A: Math. Gen. **23** (1990) 2147-2156. Printed in the UK

Kinetic Ising cellular automata models in one dimension

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Received 9 October 1989, in final form 22 January 1990

Abstract. For cellular automata versions of Glauber and Metropolis kinetic Ising models in one dimension the critical and domain growth dynamical exponents, Z_{cr} and Z_{dg}, are shown to coincide if the dynamical scaling assumption holds. Computer simulations presented yield $Z_{dg} = Z_{cr}$ equal to 2 and 1, respectively, for the Glauber and Metropolis models with checkerboard updating. The latter model with its faster relaxation is suggested as an algorithm superior to the usual Monte Carlo ones.

1. Introduction

In equilibrium statistical mechanics the study of Ising systems has played an important role with many fruitful results. In non-equilibrium statistics of Ising systems a comparably useful development followed the introduction of single spin-flip kinetic Ising models (Glauber 1963).

Among kinetic Ising models (KIMs) the one named after Glauber is the best known one, having the great advantage of being exactly soluble, though only in 1D.

Kinetic Ising models have widely been used in Monte Carlo (MC) simulations of spin systems in processes to reach thermal equilibrium (Binder 1979) as well as in non-equilibrium situations like domain growth following deep quenches (see, e.g., Gunton *et al* 1983). It is through their MC simulations that KIMs are closely related to probabilistic cellular automata (CA) since, under these circumstances, besides space and values of the spin variable time is also discrete (see Wolfram (1986) for a collection of papers). CA are also characterised by local interactions. Thus the direct construction of CA versions of KIMs is straightforward (Vichniac 1984).

There is a basic difference, however, in the way of updating spins: while it proceeds in a random sequential manner in MC procedures, in CA evolution it is synchronous. In constructing CA versions of KIMs, in order to ensure fulfilment of detailed balance with the Ising Hamiltonian, instead of fully synchronous updating an alternating updating of spins on (usually two) sublattices has been proposed (Vichniac 1984. Pomeau 1984). For details see section 2.

MC procedures introduce a basic randomness into the dynamical models under simulation. A kind of uniformity can result and several interesting features of dynamics may be lost in this way. CA representation offers a way to get rid of this uniformity effect and, as a result, simple models with a wide variety of dynamical behaviours may arise. The aim of the present paper is to illustrate this point explicitly in the cases of the two most widely used single-spin-flip KIMs, namely, the Glauber model (Glauber 1963) and the Metropolis rule (Metropolis *et al* 1953), in one dimension, where the effect is most pronounced.

0305-4470/90/112147+10$03.50

Computer simulations for the Glauber and Metropolis CA models with alternating two-sublattice updating have been carried out. Though there is no genuine phase transition in the 1D Ising model, the temperature $T=0$ acts thermodynamically as a critical point where ordering sets in, $T_{cr}=0$. Here the ferromagnetic case will be considered. We have studied (*a*) the rate at which domains of one of the degenerate stationary states of the system grow from a completely random state (quenching from temperature $T=\infty$ to $T=0$) and (*b*) the relaxation to equilibrium in the vicinity of $T=0$ as a function of temperature. Results are presented in (*a*) for the dynamic structure factor at the characteristic Bragg peak ($q=0$, in the ferromagnetic case investigated), and in (*b*) for the relaxation time of magnetisation and for the average kink density in both cases. These quantities have been found to exhibit power law behaviours (i.e. scaling) followed over about three decades in time (case (*a*)) and in $p \equiv \exp(-4J/kT)$ (case (*b*)) where J is the usual Ising coupling constant. From these results the domain growth dynamic exponent Z_{dg} in (*a*) and the critical dynamical exponent Z_{cr} in (*b*) could be deduced. For the Glauber CA model $Z_{dg}=Z_{cr}=2$ has been obtained, while for the Metropolis model $Z_{dg}=Z_{cr}=1$ resulted. The equality of Z_{dg} and Z_{cr} is argued to stem from the peculiarity of the 1D case, namely that $T_{cr}=0$, where the critically behaving coherence length $\xi \sim p^{-1/2}$ is the dominating length scale. Thus domain growth scaling reflects dynamic critical behaviour.

The reason that the two models belong to different universality classes lies in the difference of the law of motion of kinks. It is diffusive in the Glauber case while of constant velocity $v=\pm 1$ in the CA version of the Metropolis KIM. The Metropolis CA is interesting also from the point of view of computational efficiency: due to its $\tau(p) \sim p^{-1/2}$ relaxation, it is superior to Glauber MC and also to Glauber CA, for which $\tau(p) \sim p^{-1}$.

Finally, it is pointed out that Metropolis CA offer a genuine cellular automaton representation of the relaxational growth model by Krug and Spohn (1988) at $T=0$.

The paper is organised as follows. In section 2 the models, i.e. the transition rules and the way of updating, are introduced. Domain growth scaling and consequences of critical dynamics are summarised in section 3. Section 4 is devoted to the results of computer simulations and section 5 contains a discussion of the results.

2. Definition of models

Kinetic Ising models (Glauber 1969) are stochastic models in which each spin can flip or exchange with other spins in accordance with prescribed transition rates, which are so constructed that for $t \to \infty$ (t=time) the system always be found in the exact equilibrium Ising state. An infinite number of kinetic Ising models can be defined which comply with this requirement but only a few simple ones are used in practice.

Let us consider a 1D lattice with N sites, each site occupied by an Ising spin $s_i=\pm 1$. The general form of single spin-flip transition rate, W, with the restriction of up-down symmetry and of symmetrical dependence on the neighbours s_{i-1}, s_{i+1} as given by Kawasaki (1972) is

$$W(-s_i, \{s_{i'}\}|s_i, \{s_{i'}\}) = \tfrac{1}{2}\alpha[1+\delta s_{i-1}s_{i+1} - \tfrac{1}{2}\gamma(1+\delta)s_i(s_{i-1}+s_{i+1})] \tag{2.1}$$

where $\{s_{i'}\}$ denotes the states of all the spins except s_i.

The associated Hamiltonian is the ferromagnetic Ising Hamiltonian:

$$H = -(J/kT) \sum_{|i-j|=1} s_i s_j. \tag{2.2}$$

Glauber's original model (Glauber 1963) in the form most frequently used corresponds to the choice of the parameters α, δ, γ in (2.1) as follows:

$$\alpha = 1 \qquad \delta = 0 \qquad \gamma = \tanh K \qquad K \equiv 2J/kT \tag{2.3}$$

while the Metropolis rule (Metropolis *et al* 1953), usually given as

$$W_i = \min\{1, \exp[-Ks_i(s_{i-1} + s_{i+1})]\}$$

can be obtained from (2.1) with

$$\alpha = \frac{2+\gamma}{1+\gamma} \qquad \delta = -\frac{\gamma}{2+\gamma} \qquad \gamma = \tanh K. \tag{2.4}$$

Note that W_i is a transition rate and is not normalised. The basic difference between the two transition rates is that in indifferent neighbourhoods ($s_{i-1} + s_{i+1} = 0$) the probability of a spin-flip is $\frac{1}{2}$ for the Glauber model (2.3), while it is unity for the Metropolis model.

One-dimensional cellular automata are defined on a 1D regular lattice with spins s_i taking discrete values at sites i. The states are renewed at discrete time steps under the action of local rules, usually given through conditional probabilities $Q(s_i'|s_i, s_{i-1}, s_{i+1})$.

The conditional probability corresponding to (2.1) can be written as

$$Q(s_i'|s_i, s_{i-1}, s_{i+1}) = \delta_{s_i, -s_i'} W_i + \delta_{s_i, s_i'}(1 - W_i) \tag{2.5}$$

where $\delta_{x,y}$ is the Kronecker symbol. Cellular automata rules are usually applied synchronously.

In order to ensure, however, that the CA rules under consideration are in accord with the Boltzmann weight and detailed balance be satisfied with the prescribed Hamiltonian—(2.2) in our case—an updating in the checkerboard pattern has been introduced (Vichniac 1984, Pomeau 1984). Check updating in 1D means that, at time step t, every second spin, say on sites $i = 1, 3, 5, \ldots, N-1$ for t = odd, will be updated while at time step $t+1$ spins on sites $i = 2, 4, \ldots, N$ will renew their states. Thus a complete updating of the system needs two time steps. In the following we will use rules (2.1)-(2.5) and prescribe check updating.

3. Domain growth and critical dynamics

Starting from a random state and applying rule (2.5) with $\gamma = 1$ ($T = 0$) corresponds to a rapid quench of the system from a disordered state at infinite temperature into one of its competing degenerate stationary states at zero temperature. Quenches from $T = \infty$ to $0 < T < T_c$ in higher dimensions and in the framework of Ising-, Potts- and Ginzburg-Landau-type models have widely been investigated theoretically as well as via computer simulations (see Gunton *et al* (1983) for a review). Such a deep quench leads to the formation of small domains separated by domain walls. The average size of domains $L(t)$ increases in time and in the later stages of growth $L(t)$ exceeds all characteristic lengths of the system and power laws, scaling and universal behaviour can be expected to hold analogously to critical phenomena (Binder and Stauffer 1974). Dynamic scaling is usually expressed as a homogeneity relation for the structure factor

$S(q, t)$: $S(q, t) = L(t)^d \bar{S}(qL(t))$, and according to the scaling assumption domain growth is algebraic in time

$$L(t) \sim t^x \tag{3.1}$$

where d denotes dimension and $\bar{S}$ is a scaling function.

For a two-component non-conserved order parameter it is usually taken as established that $x = \frac{1}{2}$ (Allen-Cahn law (1979)), independent from T (except for the critical region).

For the models investigated here a unique peak is expected to develop at $q = 0$. Consequently, the quantity of interest is (Sadiq and Binder 1983)

$$S(0, t) = N\langle M^2(t)\rangle \sim L(t) \sim \xi_s(t) \tag{3.2}$$

where $M(t) = (1/N)\,\Sigma_i s_i$ and $\langle\ \rangle$ denotes averaging over all initial states. $\xi_s(t)$ is the dynamic coherence length of the system defining the domain growth dynamic exponent, Z_{dg}, as

$$\xi_s(t) \sim t^{1/Z_{dg}}. \tag{3.3}$$

Another quantity, the average density of kinks:

$$\bar{n}(t) = \left\langle \frac{1}{N} \sum_i \tfrac{1}{2}(1 - s_i s_{i+1}) \right\rangle \tag{3.4}$$

also defines a relevant length measure (Sadiq and Binder 1983)

$$\bar{n}(t) \sim 1/\xi_n(t) \sim t^{-y}. \tag{3.5}$$

Whenever scaling is satisfied with a single characteristic length, $y = x$ is to be expected.

The one-dimensional case is specific as compared to higher-dimensional ones because the whole temperature region of the ordered phase shrinks to the single point $T = 0$ which is the critical point too. This has the consequence that, though $L(t) > \xi$ is not fulfilled for any time, characteristic domain growth proceeds and scaling is expected to be governed by critical dynamics (ξ is the correlation length in the 1D Ising system).

The domain walls separating the ferromagnetic domains: kinks (up-down boundaries) and antikinks (down-up boundaries) move diffusively and annihilate pairwise upon meeting: kink + antikink → 0 in case of Glauber dynamics (figure 1(*a*)). Diffusion-limited annihilation is a well studied problem of mathematics (Griffeath 1979). For the CA version of the Glauber model Domany and Kinzel (1984) have derived the theoretical expression for $\bar{n}(t)$, namely

$$\bar{n}(t) = (8\pi Dt)^{-1/2} \qquad t \to \infty \tag{3.6}$$

Figure 1. Evolution of the (*a*) Glauber CA and (*b*) Metropolis CA from random initial configuration through the first 50 time steps. Horizontal axis: space coordinate, vertical axis: time, increasing downwards. Black regions: $s_i = +1$; white regions: $s_i = -1$.

where D is the diffusion constant. $D = \frac{1}{2}$ results in the standard annihilative RW problem, i.e. when particles move at each time step to the right or to the left with probability $\frac{1}{2}$, without any halting time.

For one-dimensional Ising statics the singularities of thermodynamical quantities when $T \to 0$ are essential ones. Nevertheless some of the critical exponents can be defined if we use $p \equiv \exp(-2K)$, $K \equiv 2J/kT$, instead of T. From exact solution of the 1D Ising model (see, e.g., Stanley, 1971) the coherence length, ξ, and the static susceptibility, χ, are known to behave like

$$\xi \sim p^{-\nu} \qquad kT\chi \sim p^{-\bar{\gamma}} \qquad \bar{\gamma} = \nu = \tfrac{1}{2}. \tag{3.7}$$

Moreover, from the result for the equilibrium correlation $\langle s_i s_{i+1} \rangle = \tanh(K/2) \approx 1 - 2p^{1/2}$, the equilibrium density of kinks, $\bar{n}(p)$, is obtained, using (3.4), as

$$\bar{n}(p) = p^{\alpha_n} \qquad \alpha_n = \tfrac{1}{2}. \tag{3.8}$$

Concerning dynamic critical scaling (Ferrell *et al* 1967, Halperin and Hohenberg 1967) its predictions for the dynamic structure factor at $q = 0$ and for the kink density can be given as follows.

Supposing that there is a single characteristic time in the system, $\tau \sim p^{-\nu Z_{cr}}$, in the limit of vanishing p, $S(0, p, t)$ can be written as

$$S(0, p, t) = t^{x} g_s(\varepsilon) \qquad \varepsilon = tp^{\nu Z_{cr}} \tag{3.9}$$

where Z_{cr} is the critical dynamical exponent and the scaling function $g_s(\varepsilon) = \text{constant}$ for $\varepsilon \to 0$. For $\varepsilon \gg 1$, however, $g_s(\varepsilon)$ is supposed to behave like $g_s(\varepsilon) = \varepsilon^{-x} \times \text{constant}$. In this limit $S(0, p, t) \sim p^{-\bar{\gamma}}$ has to hold; thus

$$x = \bar{\gamma}/\nu Z_{cr} \tag{3.10}$$

arises.

Similar relations can also be written for $\bar{n}(p, t)$:

$$\bar{n}(p, t) = t^{-y} g_n(\varepsilon) \qquad \varepsilon \to 0 \tag{3.11}$$

with $g_n(0) = \text{constant}$. For $\varepsilon \gg 1$ $g_n(\varepsilon) = \varepsilon^{y} \times \text{constant}$ yields the expected $\bar{n}(p, t) \sim p^{\alpha_n}$ with the second scaling law†

$$y = \alpha_n/\nu Z_{cr}. \tag{3.12}$$

Using (3.7) and (3.8) we get

$$x = y = 1/Z_{cr} \tag{3.13}$$

It is an important feature of the 1D case that (3.9) and (3.11) refer to the same quantities as (3.1) and (3.5), respectively. Z_{cr} is to be determined via the linear relaxation time

$$\tau(p) \sim p^{-\nu Z_{cr}} = p^{-Z_{cr}/2}. \tag{3.14}$$

In the case of the Glauber model the exact solution yields (Glauber 1963)

$$\tau(p) = \frac{1}{1 - \tanh K} \approx \frac{1}{2p}. \tag{3.15}$$

† It is to be noted that in higher dimensions the second scaling law contains the critical exponent α of the specific heat as $1 - \alpha$ (Sadiq and Binder 1983) instead of α_n. The reason is that, in 1D, α is negative and the scaling behaviour is carried by the next-to-leading-order term.

In the following we will present results of numerical simulations for $S(0, t) \equiv S(0, 0, t)$ and $\bar{n}(t) \equiv \bar{n}(0, t)$ by starting from a completely random initial state and applying (2.1) with $\gamma = 1$ ($p = 0$) while $\tau(p)$ and $\bar{n}(p) \equiv \bar{n}(p, \infty)$ will be measured under the condition of a pure initial state (say, with $M(t = 0) = 1$; M is the magnetisation density) using (2.1) for various values of p, $p \neq 0$, small.

4. Results of computer simulations

4.1. The Metropolis model

The Metropolis rule (2.1) with (2.4) has no inherent randomness at $T = 0$. When applied with random sequential updating as in a Monte Carlo simulation, it leads to the same result as the Glauber model.

The same rule with check updating produces the spacetime picture as shown in figure 1(*b*) if we start from a random initial state with zero average magnetisation.

Domains of pure up-spin and down-spin sites are separated by kinks (antikinks) moving with maximal velocity $v = +1$ (-1) to the right (left). Upon meeting, a kink and an antikink annihilate (particles cannot cross each others' trajectories). As $t \to \infty$, provided the initial state has been prepared so that equal number of kinks and antikinks are present, one of the degenerate pure states will result. Starting from a random initial distribution of 1 and -1 at $t = 1$, in the spirit of check-updating as described in section 2, the rule, on the particle level (i.e. in terms of kinks and antikinks, instead of spins) can be formulated as follows: a domain boundary will move to the right provided it is situated at $t = 1$ between even-odd indexed sites (i = even, $i + 1$ = odd) while in the case of a domain boundary between odd-even indexed sites (i = odd, $i + 1$ = even) an antikink is emitted. By interchanging odd and even, an equivalent situation arises, of course. From then on the whole process is deterministic. The only random element is contained in the averaging over all possible initial states.

In simulating a quench from $T = \infty$ to $T = 0$ we have started from disordered configurations of zero average magnetisation and averaging over many (100–200 in practice) independent initial states. Here and in all further CA simulations, periodic boundary conditions have been applied and the timescale was such that the two steps of check updating counted as two units of time.

For $S(0, t)$ and $\bar{n}(t)$ two characteristic power laws indicating scaling have been observed (figure 2):

$$S(0, t) \sim a_x t^x \qquad a_x = 0.96 \pm 0.05 \qquad x = 0.98 \pm 0.05$$

$$\bar{n}(t) = a_y t^{-y} \qquad a_y = 0.285 \pm 0.005 \qquad y = 0.49 \pm 0.01.$$

Our result gives, according to (3.2), $S(0, t) \sim \xi_s(t) \sim t$, $\xi_s(t)$ being the dynamical coherence length of the problem, determined by the linear law of motion of kinks and antikinks: $x = vt$, $v = \pm 1$. From (3.3) $Z_{dg} = 1$ follows.

As to the result for $\bar{n}(t)$, it is in good agreement with (3.6), the expression for diffusion-limited annihilation, with $D = \frac{1}{2}$. The rule being wholly deterministic, the source of randomness lies solely in the initial state. In an equivalent problem of the ideal gas of point particles A performing free motion with velocities $v = \pm 1$ on a line and with annihilation kinetics $A_{v=1} + A_{v=-1} \to 0$, Elskens and Frisch (1985) have been the first to derive $\bar{n}(t)$, starting from an initial state in which particle positions have a Poisson distribution. The length measure $\xi_n(t)$ of (3.5) is thus not an intrinsic one; it characterises the initial state.

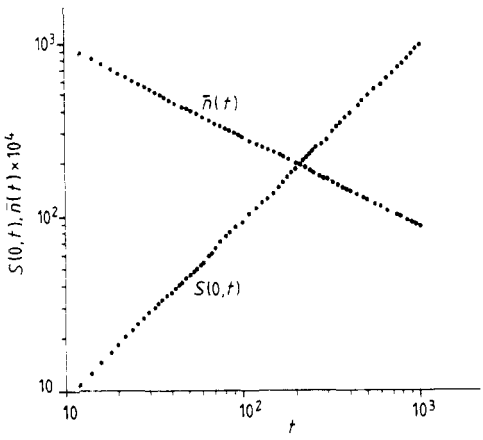

Figure 2. Results of computer simulation for $S(0, t)$ and $\bar{n}(t)$ in the case of the Metropolis CA model. $N = 2 \times 10^4$ and averaging over 150 independent initial configurations has been carried out.

At $T \neq 0$ ($p \neq 0$) kink-antikink pairs can also be created at positions belonging to domain interiors. According to the rule on the particle level, as described before, kink-antikink pairs are allowed to arise only so that the seeds of new-born phases lie on sites satisfying $i + t =$ even. The spacetime picture of the process at $p = 0.01$ is shown in figure 3.

As time goes on creation and annhihilation of kink-antikink pairs reaches equilibrium, and for $t \to \infty$ some equilibrium kink density, $\bar{n}(p)$, arises for which (3.8) holds as an exact result. In measuring the critical slowing down, the relaxation time $\tau(p)$ was defined as the average time during which an initial pure state with magnetisation density $M(0) = 1$ has decayed to a state with $M(\tau) = 0.5$.

A lattice of $N = 10^4$ sites was used and averages over 100 independent runs have been performed. The results are (figure 4)

$$\tau(p) = a_\tau p^{-\alpha_\tau} \qquad a_\tau = 1.05 \pm 0.1 \qquad \alpha_\tau = 0.51 \pm 0.02$$

$$n(p) = a_n p^{\alpha_n} \qquad a_n = 0.95 \pm 0.1 \qquad \alpha_n = 0.49 \pm 0.02.$$

From the result obtained for $\tau(p)$, namely that $\alpha_\tau = \frac{1}{2}$ within numerical error, using (3.14), $Z_{cr} = 1$ follows. This finding, together with establishing $Z_{dg} = Z_{cr}$ for the Metropolis CA model, constitutes the main result of this paper. The good agreement of the simulations for $\bar{n}(p)$ with (3.8) is regarded as a check.

Figure 3. Spacetime evolution of Metropolis CA with $p = 10^{-2}$ for the first 50 time steps, starting from an ordered state with $|M(0)| = 1$.

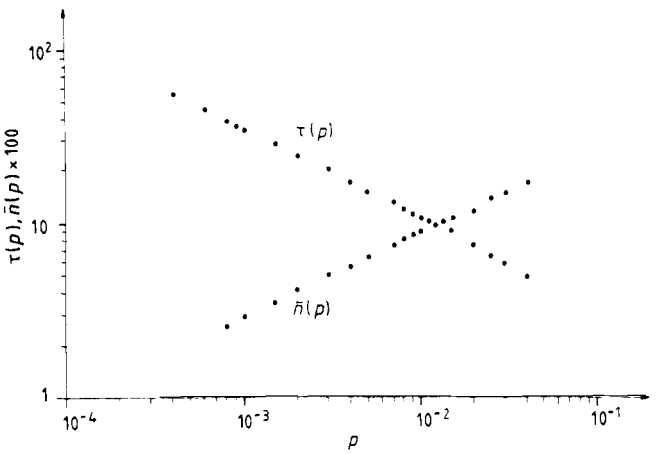

Figure 4. Results of computer measurements of $\bar{n}(p)$ and $\tau(p)$ for the Metropolis CA at $p \neq 0$.

4.2. The Glauber model

Though the Glauber model is exactly solvable in 1D and $\bar{n}(t)$ has also been derived for its CA version, as mentioned in the previous subsection, the explicit result for $S(0, t)$ appears not to be available. Thus, for the sake of completeness, we provide here results of computer simulations also for the Glauber model.

The results for $S(0, t)$ and $\bar{n}(t)$ are as follows (figure 5):

$$\bar{n}(t) = a_y t^{-y} \qquad a_y = 0.282 \pm 0.002 \qquad y = 0.50 \pm 0.01$$

$$S(0, t) = a_x t^{x} \qquad a_x = 2.2 \pm 0.2 \qquad x = 0.49 \pm 0.03.$$

The $t^{1/2}$ behaviour of $S(0, t)$ is in agreement with the Allen-Cahn (1979) law. Moreover, according to (3.2) and (3.3) it gives the domain growth dynamic exponent as $Z_{dg} = 2$, within numerical error. The result for $\bar{n}(t)$ agrees with (3.6) with $D = \frac{1}{2}$ again.

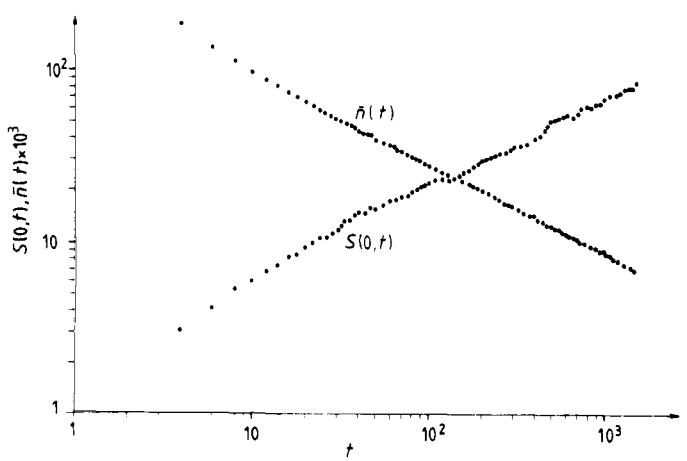

Figure 5. Results of computer simulation for $S(0, t)$ and $\bar{n}(t)$ in the case of Glauber CA. The lattice size is $N = 10^4$, and averaging over 200 independent random initial states with zero average magnetisation has been performed.

In the simulation of critical slowing down for the Glauber model the relaxation time $\tau(p)$ was defined as for the Metropolis CA model. We have obtained:

$$\tau(p) \propto a_\tau p^{-\alpha_\tau} \qquad a_\tau = 0.5 \pm 0.01 \qquad \alpha_\tau = 1.0 \pm 0.02$$

$$\bar{n}(p) \propto a_n p^{\alpha_n} \qquad a_n = 0.94 \pm 0.05 \qquad \alpha_n = 0.505 \pm 0.01.$$

The agreement with the exact results (3.15) and (3.8) is satisfactory. We conclude also in the Glauber case that $Z_{dg} = Z_{cr}$.

5. Summary and discussion

In this paper cellular automata versions of two most frequently used single-spin-flip kinetic Ising models have been investigated in one dimension by performing computer measurements under the conditions of (*a*) rapid quench to $T = 0$ and (*b*) critical slowing down in the vicinity of $T = 0$ as a function of time and temperature, respectively.

For the Glauber CA model we have found the same behaviour—as could be expected—as from MC simulations. The circumstance that checkerboard updating does not result in any change can be traced back to the probabilistic nature of the rule, especially to the inherent randomness present through the component $Q(1|-1, 1, -1) = \frac{1}{2}$ of the transition probability determining kink kinetics.

The Metropolis model, however, with check updating belongs to a different universality class: the dynamical exponents measured under conditions (*a*) and (*b*) above and Z_{dg} and Z_{cr} both equal unity. This value reflects the linear law of motion of the kinks $v = \pm t$ persisting for $T \neq 0$, too. The reason lies in the deterministic and Class 1 nature† of the Metropolis CA; that part of the transition rule ($Q(1|-1, 1, -1) = 1$ determining kink motion is deterministic at all temperatures, and not influenced by the random nature of the kink-antikink creation ($Q(-1|1, 1, 1) = p$). Check updating, naturally, does not introduce any randomness. (Changing to random sequential updating, the universality class changes, $Z_{dg} = Z_{cr} = 2$ are obtained.) Dynamic critical scaling, which leads to $Z_{dg} = Z_{cr}$, is partly violated, namely in the $n(t) \sim t^{-1/2}$ behaviour of the kink density under condition (*a*), as $n(t)$ reflects the scaling properties of the ensemble of initial (random) states.

The Metropolis CA model is equivalent—concerning kink dynamics—under condition (*a*) to a deterministic growth model proposed by Krug and Spohn (1988). The surface steps in their model correspond to the kinks of the Metropolis CA. Using scaling arguments and also calculating the two-surface-step correlation function exactly, these authors have found $\xi = 2t$ for the intrinsic coherence length and thus $Z_{dg} = 1$ for the non-equilibrium dynamic exponent, in accord with our result.

Concerning CA 184 (in Wolfram's (1984) coding) claimed to be equivalent, on the particle level, to their growth model by Krug and Spohn (1988), the following problems arise. In this CA spin has values 1, 0 and updating is synchronous, the pure states are period-two antiferromagnetic ones; kinks are two neighbouring 0 while antikinks are two neighbouring 1. Besides these, however, point defects (odd numbers of adjacent 1 or 0) and further particles (even numbers of adjacent 0) and antiparticles (even numbers of adjacent 1) with complicated reactions (e.g. $0000 + 11 \rightarrow 000$) are also

† Class 1 means that the evolution of the CA from a random initial state leads to a fixed point for $t \rightarrow \infty$ (Wolfram 1984). In the deterministic but chaotic (Class 3) automaton number 18 (in Wolfram's (1984) coding), $Z_{dg} = 2$ has resulted from computer simulation.

possible, though with lesser probability. A further drawback of using this CA as a simple growth model is that its extension to finite 'temperatures' ($p \neq 0$) by the natural choice $Q(1|111)=1$, $Q(1|110)=0$, $Q(1|011)=1-p$, $Q(1|101)=1$, $Q(1|010)=1$, $Q(1|100)=p$, $Q(1|001)=0$, $Q(1|000)=0$ gives rise to kink + antikink pair creation as a second-order effect; to first order in p point defects are created.

The Metropolis CA, however, under quench conditions, is a genuine CA equivalent of the above-mentioned growth model.

Finally, through the result obtained in this paper, namely that for the Metropolis CA, $\tau(p) \sim p^{-1/2}$, while for the Glauber model $\tau(p) \sim p^{-1}$, the Metropolis CA offers a more efficient way of computer simulation and a quicker relaxation algorithm than usually applied.

It would be worth extending the investigations to two dimensions. In 2D, however, using CA versions of KIM $S(q, t)$ has been studied by Viñals and Gunton (1986) with a result indicating the unusual behaviour of the Metropolis rule in 2D also (they reported a quicker than power-law dependence for $S(0, t)$) but, to our knowledge, no measurements concerning critical properties of this cellular automaton are available in 2D.

Acknowledgments

The author is indebted to J Kertész for calling her attention to the paper by Krug and Spohn and for illuminating discussions. Useful remarks from M Droz, Z Rácz and P Szépfalusy are gratefully acknowledged.

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